Further Maths for Engineering Technicians

1. The displacement x meters of a mass from a fixed point about which it is oscillating is given by $x = 3.2\cos 20\pi t + 2.4\sin 20\pi t$ where t is the time in seconds

a. Express the displacement in the form $x = A\sin(\omega t + Ø)$

Oscillation amplitude is

$$\sqrt{3.2^2 + 2.4^2} = \sqrt{16} = 4$$

The frequency of oscillations is $f = 20\pi$. Period of oscillations is

$$T = \frac{2\pi}{f} = \frac{2\pi}{20\pi} = 0.1\ s$$

Therefore, we get

$$4\left(\frac{3.2}{4}\cos 20\pi t - \frac{2.4}{4}\sin 20\pi t\right)$$

Here

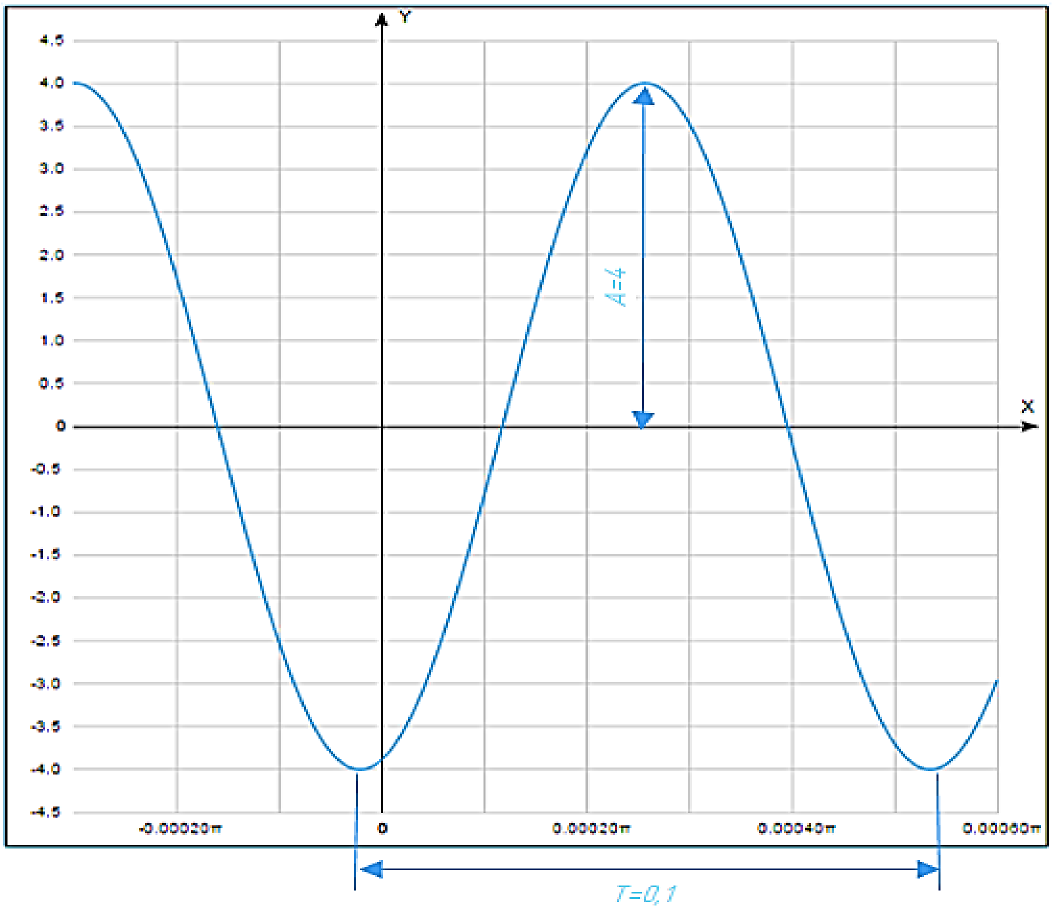
$$\frac{3.2}{4} = \sin Ø; \quad -\frac{2.4}{4} = \cos Ø$$

The angle is in the 2nd quadrant

$$Ø = 2\pi - \sin^{-1}\frac{3.2}{4} = 127^\circ$$

$x = 4\sin(20\pi - 127)$

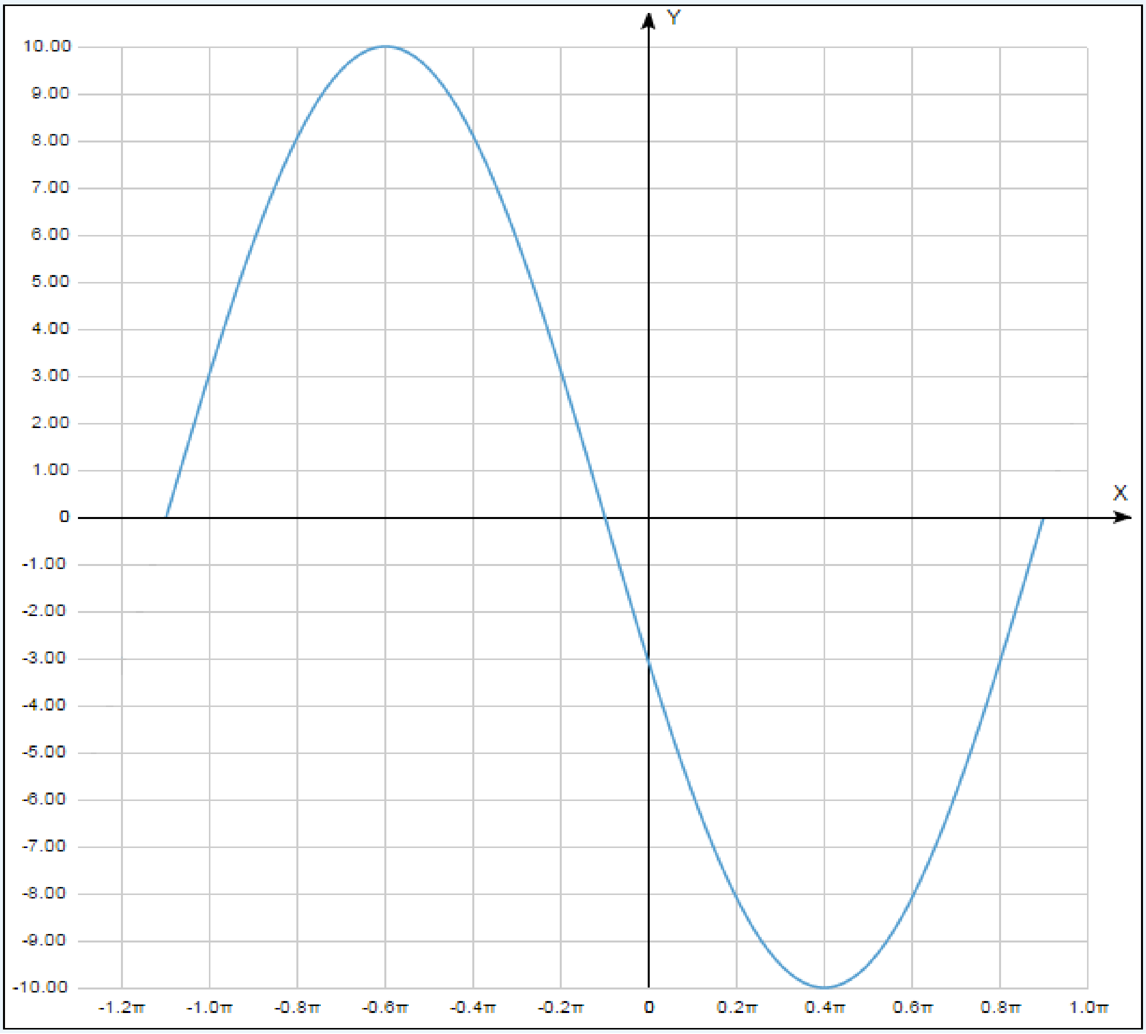
b. Draw one cycle of the waveform and label amplitude and periodic time:



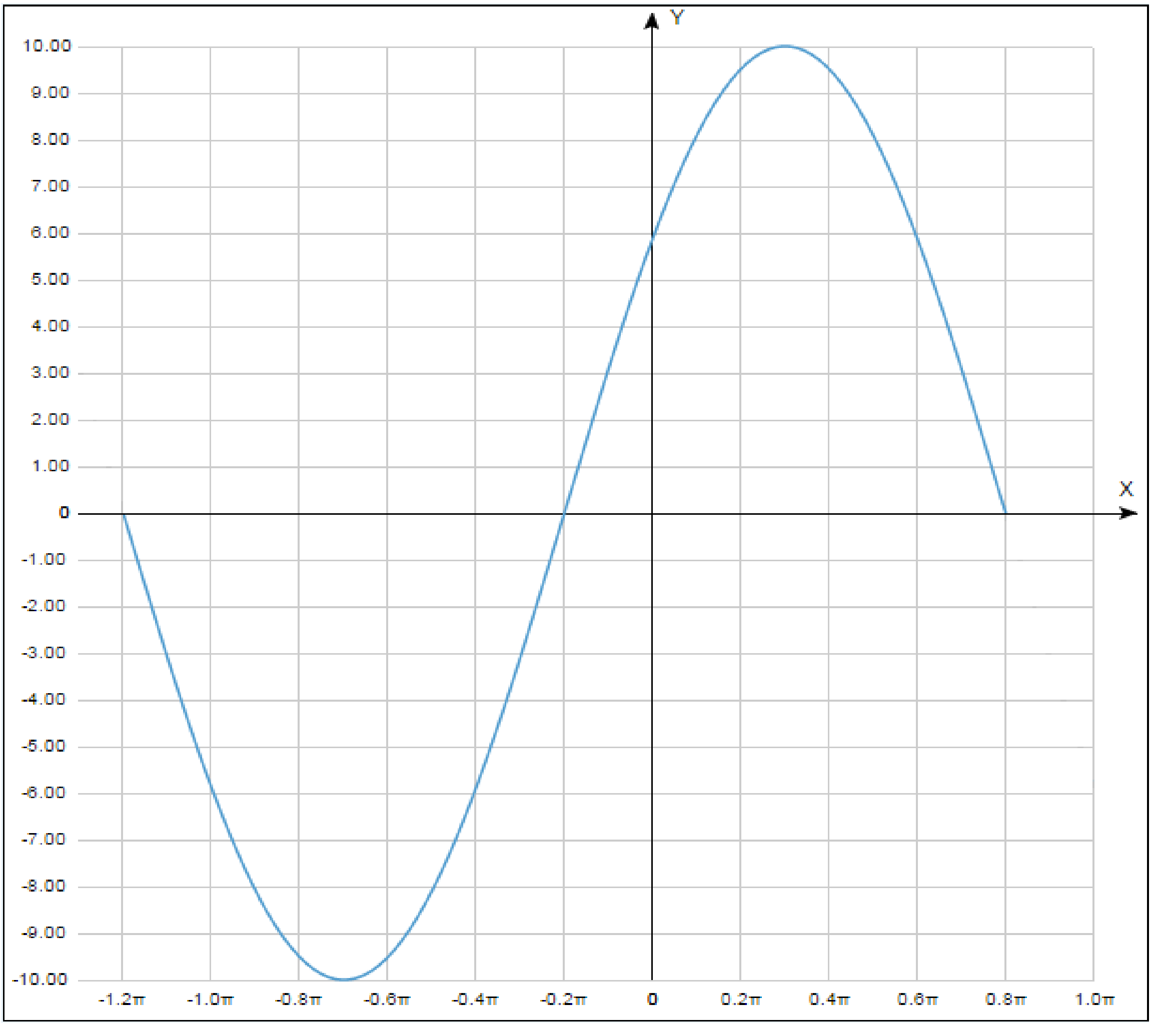
c. Amplitude is the maximum displacement of the body, i.e. the distance of the wave vibration (oscillation), periodic time it takes for one vibration cycle for a positive and negative amplitude; frequency is the amount of cycles per 1 second.

2. On one graph draw one complete cycle for the following waveforms:

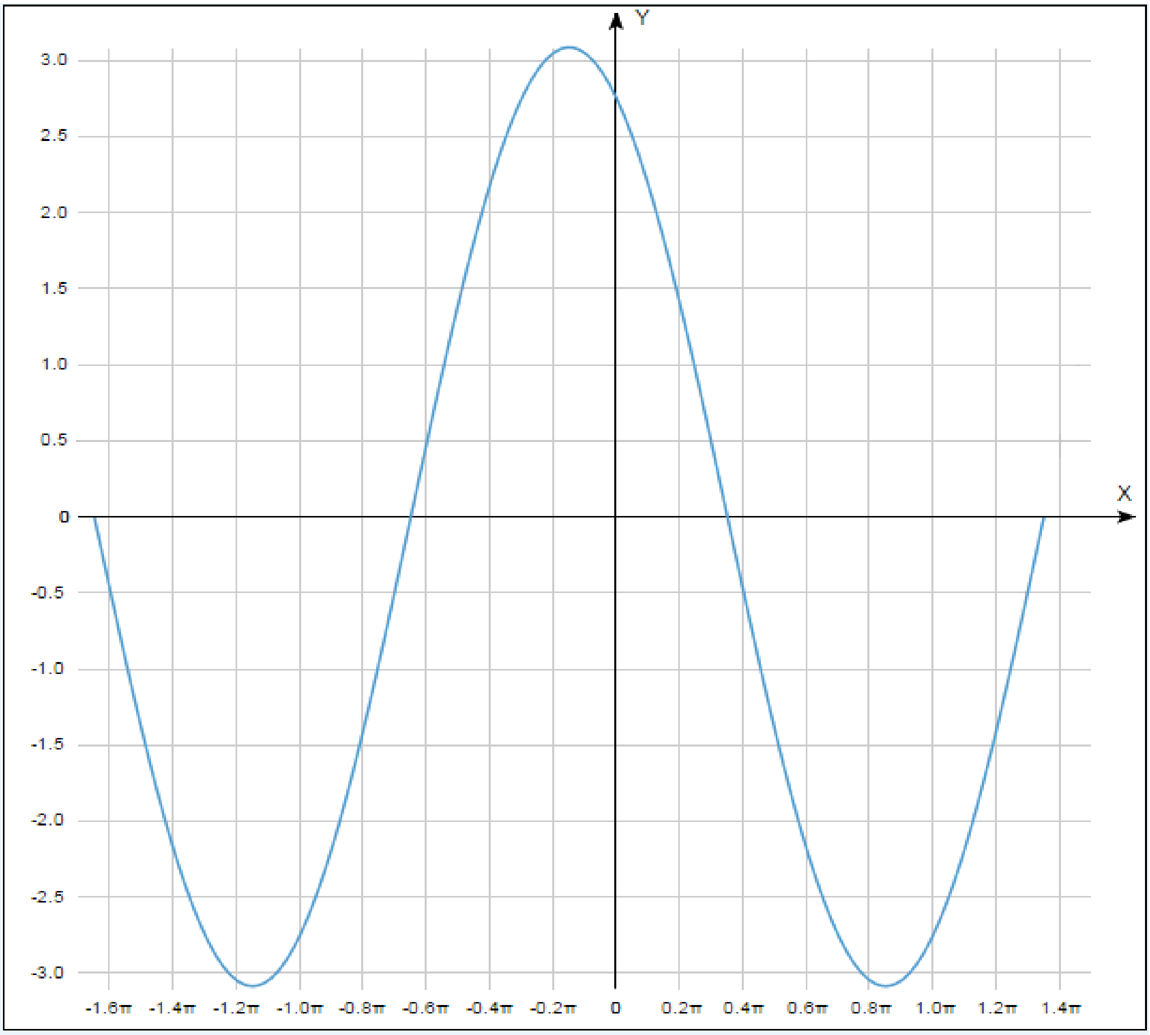
I. $10\sin(x + \frac{\pi}{3})$



II. $10\sin\left(x + \frac{2\pi}{3}\right)$



III. $10\sin(x + \frac{\pi}{3}) + 10\sin(x + \frac{2\pi}{3})$



a. Compare waveform III. to $\cos x$

These two graphs are very similar. However, it can be seen that the presented graph is shifted a little right along the **x-axis** in comparison with $\cos x$ graph since $\cos x$ highest point is (0,1) and lowest (π,1).

b. Use compound angle formulae to prove your observation.

$$10\sin\left(x+\frac{\pi}{3}\right)+10\sin\left(x+\frac{2\pi}{3}\right)=10(\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x+\frac{2\pi}{3}\right))=2\sin\frac{(x+\frac{\pi}{3})+(x+\frac{2\pi}{3})}{2}\bullet\cos\frac{(x+\frac{\pi}{3})+(x+\frac{2\pi}{3})}{2}=$$

$$=2\sin\frac{2x+\pi}{2}\bullet\cos\left(-\frac{\pi}{6}\right)=2\sin\left(x+\frac{\pi}{2}\right)\bullet\frac{\sqrt{3}}{2}=$$

$$=\sqrt{3}\sin\left(x+\frac{\pi}{2}\right)$$

Applying another formula:

$$\sqrt{3}\sin\left(x+\frac{\pi}{2}\right)=\sqrt{3}\left(\sin x\bullet\cos\frac{\pi}{2}+\cos x\bullet\sin\frac{\pi}{2}\right)$$

The first term is equal to zero since $\cos\frac{\pi}{2}=0$, as

$\sin\frac{\pi}{2}=1$ the final result is $\sqrt{3}\cos x$. These formulae prove that the graph of this function differs from $\cos x$ and is shifted along x-axis.

3.

a. Simplify:

$$\cos 6\omega t\ \cos 4\omega t + \sin 6\omega t\ \sin 4\omega t =$$

$$= \frac{\cos(6\omega t - 4\omega t) + \cos(6\omega t + 4\omega t)}{2} +$$

$$+ \frac{\cos(6\omega t - 4\omega t) - \cos(6\omega t + 4\omega t)}{2} =$$

$$= \frac{\cos 2\omega t + \cos 10\omega t}{2} + \frac{\cos 2\omega t - \cos 10\omega t}{2} =$$

$$= \frac{1}{2}(\cos 2\omega t + \cos 10\omega t + \cos 2\omega t - \cos 10\omega t) =$$

$$= \frac{1}{2} \bullet 2\cos 2\omega t = \cos 2\omega t$$

b. Determine all possible solutions for the following equation in the range:

$$0 \le \text{Ø} \le 2\pi$$

$$2\sec^2\text{Ø} + 5\tan\text{Ø} = 3$$

$$2(1 + tan^2 x) + 5\tan x = 3$$

$$2\, tan^2 x + 5\tan x = 1$$

Here, it is convenient to make a replacement . We get a simple quadratic equation:

$$2y^2 + 5y = 1$$

$$y_{1,2} = \frac{-5 \pm \sqrt{2}}{2}$$

$$\tan x_{1,2} = \frac{-5 \pm \sqrt{2}}{2}$$

1) $x_1 = \tan^{-1} \frac{-5 + \sqrt{2}}{2} + \pi n \approx \tan^{-1}(-3.2) + \pi n, n \in Z;$

$x_1 = -0.4\pi + \pi n, n \in Z$

For the given interval, the possible solution is 0.4π and 1.4π

2) $x_1 = \tan^{-1} \frac{-5 - \sqrt{2}}{2} + \pi n \approx \tan^{-1}(-2.1) + \pi n, n \in Z;$

$x_1 = -0.36\pi + \pi n, n \in Z$

For the given interval the possible solution is 0.36π and 1.36π

c. Using the exponential representation of hyperbolic functions, simplify:

$$-3 + \frac{3}{2} cosh^2 A = -6 + 3\,\frac{\cosh 2A + 1}{2} = -11 + 3\cosh 2A =$$

$$= -11 + 6\left(\frac{e^A + e^{-A}}{2} \bullet \frac{e^A - e^{-A}}{2}\right) = -44 + 6\left(e^{2A} - e^0 + e^0 - e^{-2A}\right) =$$

$$= -44 + 6\left(e^{2A} - e^{-2A}\right) = e^{2A} - e^{-2A} - 7.3$$

4. A canal of width 25 metres is taken through a cutting as shown below. A cable is to span the cutting such that at its lowest point the catenary has a clearance of 45 metres vertically above the canal. The heights of the 2 supporting pylons A & B are 48m and 59m respectively above the water level.

Determine:

a. The equation for the curve taken by the cable.

Here, we have 3 points, via which passes the cable. The common form for the curve equation by three points is a follows:

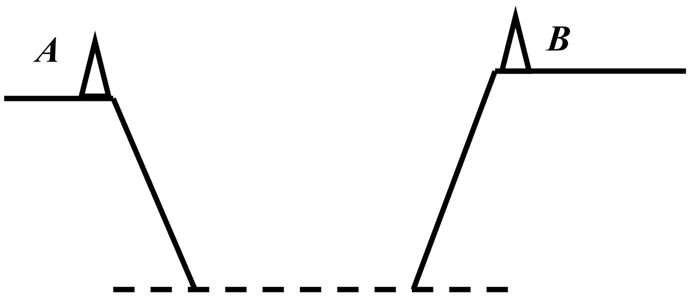
$$[x,y] = (1 - t)^2 \bullet P_0 + (1 - t)tP_1 + t^2 P_2$$

Where P_0, P_1, P_2 are points 48, 45 and 59 meters respectively. Therefore the equation for the cable curve is:

$$(1 - t)^2 48 + (1 - t)\, 45t + t^2 59 = (1 - 2t + t^2)48 + 45t - 45t^2 +$$

$$+ 59t^2 = 48 - 96t + 48t^2 + 45t - 45t^2 + 59t^2 = 62t^2 - 51t + 48$$

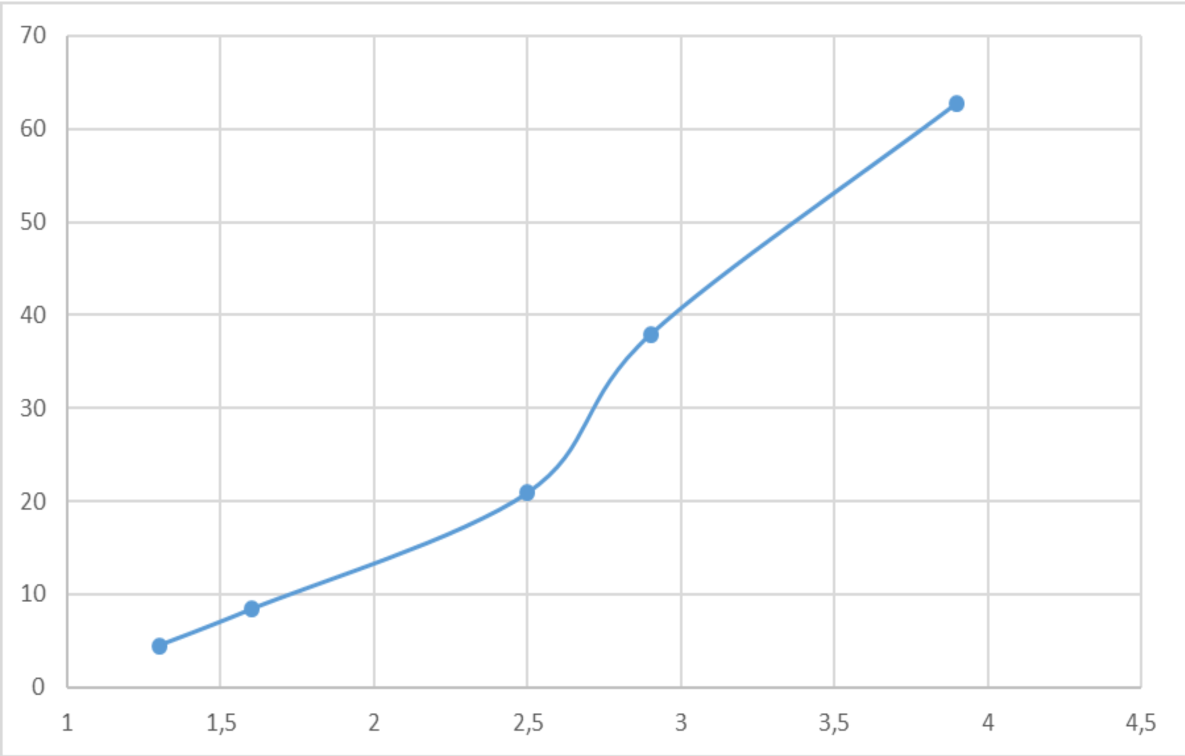
b. The horizontal difference between the pylons.



5. In an experiment the following results were obtained.

x	1.3	1.6	2.5	2.9	3.9
y	4.5	8.4	20.9	37.9	62.8

Draw a graph to determine that the readings follow a law corresponding to $y = Ax^b$ and determine the values for constants A and b.



The graph shows that readings relatively correspond to the given law. The obtained graph looks like the cubic function since it is almost symmetrical in relation to point (2.75; 30) and without the first point.

Therefore, the constants for the given law are: $b = 3$; $A = \dfrac{y}{x^3}$.

Substitution know values for y and x we get: $A = \dfrac{8.4}{1.6^3} = 2.05$

The function for the obtained data is: $y = 2.05x^3$